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DETERMINING VELOCITIES BY PROPAGATION

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ABSTRACT

A velocity propagation technique is described that determines velocity vectors at the points of a contour, based on the velocities at the endpoints of the contour and the normal components of velocity along the contour.

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1. Introduction

Image motion can be described by a velocity or optical flow field $V(x,y,t)$ which gives the direction and speed of movement of each point (x,y) in an image at time t. A variety of algorithms have been proposed for determining this optical flow velocity field. One class of such algorithms is based on a simple linear relationship between spatial and temporal intensity gradients [1,2,3]. These gradients are related to the optical flow by the following equation:

$$I_x u + I_y v + I_t = 0 \tag{1.1}$$

or

$$V_n = - I_t/|\nabla I| \tag{1.2}$$

where I_t is the temporal intensity change at (x,y); I_x and I_y are the spatial intensity changes along the x and y axes at time t; $|\nabla I|$ is the magnitude of the spatial intensity gradient at that point ($|\nabla I| = \sqrt{I_x^2+I_y^2}$); V_n is the normal component of the velocity at that point (along the intensity gradient); and u and v are the components of the velocity in the x and y directions.

However, it is impossible to determine both u and v or both V_n and V_t (where V_t is the tangential component) from the single constraint of eq (1.1) or eq (1.2). So certain further assumptions about the organization of the velocity field have to be made. Horn and Schunck [3], for example, proposed the assumption that the velocity varies smoothly. However, along boundaries between objects moving with different velocities, the resulting velocity field is unreliable, because this smoothness assumption becomes invalid.

A similar problem (the so-called aperture problem) has been studied by Batali and Ullman [4] in connection with detecting motion along image contours (zero-crossings of Laplacians in their work) . The aperture problem is that if motion is detected by an element which is small compared to the overall contour, then the only motion information that one can obtain is the component perpendicular to the local orientation of the contour. Motion along the contour cannot be determined. They suggest that in the case of translation, the overall motion can be recovered by combining the local motion constraints. Their method appears to rely heavily on the assumption that the motion is a simple translation in the image plane.

Yachida [5] proposed an iterative scheme for propagating the velocity from some prominent points with given initial velocity estimates. This scheme was also based on the smoothness assumption of the velocity field.

In this paper we present a local constraint between the velocity vectors at the two ends of a small line segment. This constraint is based on the assumption that all motions are rigid, and it is used to derive a propagation procedure which can assign velocity vectors to all points on an image contour, based on the velocity vectors at the endpoints of the contour, and on the normal components of the velocity vectors along the contour. Since the method does not combine information across an edge, it should succeed in just those cases where a method such as that of Horn and Schunk [3] would have difficulty.

2. Theory

2.1 The local constraint and the propagation formula

Suppose the velocity vectors $\underline{V_0}, \underline{V_n}$ at the ends of a contour A_0A_n are known (see Figure 1). Consider a small line segment dS along the contour A_0A_1. Assuming that the motion is a rigid motion and is small relative to the quantization grid and contour curvature, the component V_{0S} of V_0, the motion at A_0, parallel to A_0A_1 must equal the parallel component V_{1S} of the velocity $\underline{V_1}$ at A_1:

$$V_{0S} = V_{1S} \tag{2.1a}$$

or

$$\underline{V_0} \cdot \overline{dS} = \underline{V_1} \cdot \overline{dS}. \tag{2.1b}$$

where $\underline{V_0}$ and $\underline{V_1}$ are the velocity vectors at the two ends of the line dS, and $\overline{dS}$ is the unit vector along dS, the vector joining A_0 to A_1. Rewriting this local constraint (eq. 2.1b) into component form, we obtain

$$\begin{aligned} \underline{V_0} \cdot \overline{dS} &= (V_{1n}\overline{n} + V_{1t}\overline{t}) \cdot \overline{dS} \\ &= V_{1n}\overline{n} \cdot \overline{dS} + V_{1t}\overline{t} \cdot \overline{dS} \end{aligned} \tag{2.2}$$

where V_{1n} and V_{1t} are the normal component and the tangential component of the velocity vector $\underline{V_1}$ respectively, and $\overline{n}$ and $\overline{t}$ are the unit vectors in the normal and tangent directions of the contour at A_1. From Figure 1, we see that

$$V_{0S} = V_{1t} \sin\alpha + V_{1n} \cos\alpha$$

Thus, the tangential component is

$$V_{1t} = (V_{0S} - V_{1n} \cdot \cos\alpha)/\sin\alpha \qquad (2.3)$$

where α is the angle between the unit vector $\overline{dS}$ and the normal vector $\bar{n}$ at the point A_1. We also have $\gamma=\beta-\alpha$, where β is the angle between the x-axis and the normal vector $\bar{n}$, and γ is the angle between the x-axis and the line segment dS.

We can propagate the velocity along a contour using eq.(2.3), because the first projection V_{0S} is known after the previous propagation and the normal component V_{1n} can be computed by, e.g., the methods discussed in [3] or [4]. A procedure similar to the one described in [3] was used for computing normal velocity components in the experiments described in the following section. Once V_{1n} is computed, $\underline{V_1}=V_1 e^{j\theta}$ can be obtained because

$$V_1 = \sqrt{V_{1n}^2+V_{1t}^2} \qquad (2.4)$$

$$\theta = \beta - \arctan V_{1t}/V_{1n}$$

2.2 Error analysis and a correction technique

From eq. (2.3) the new estimate of the tangent component V_{1t} is based on the previous projection V_{0S} and on the normal component V_{1n} at the current propagation point. Differentiating this equation we obtain

$$dV_{1t} = \frac{\partial V_{1t}}{\partial V_{0S}}\, dV_{0S} + \frac{\partial V_{1t}}{\partial V_{1n}}\, dV_{1n} + \frac{\partial V_{1t}}{\partial \alpha}\, d\alpha$$

$$= \frac{1}{\sin\alpha}\, dV_{0S} + \cot\alpha\, dV_{1n} + \frac{(V_{1n} - V_{0S}\cos\alpha)}{\sin^2\alpha}\, d\alpha \qquad (2.5)$$

Note that the error in V_{1t} depends on the error in the previous projection (dV_{0S}), the error in the normal component V_{1n} at the current propagation point (dV_{1n}), and the error in the measurement of the angle α ($d\alpha$).

The result of these various errors is that when the propagation reaches A_n, the velocity vector attributed to A_n by the propagation procedure will differ from the velocity vector originally computed at A_n. Therefore, at the point A_n we compute the error between the propagation velocity estimate $\underline{V}'_n$ and the original velocity vector $\underline{V}_n$:

$$\underline{\Delta V}_n = \underline{V}_n - \underline{V}'_n$$

If this error is less than some tolerance, then this propagation procedure is stopped at point A_n; otherwise a correction procedure is applied. If we consider the error ΔV_n as having been accumulated in the previous n steps, then the average velocity error in one step is

$$\underline{V_e} = \underline{\Delta V_n}/n$$

so we have $m \cdot \underline{V_e}$ as the velocity error at the m^{th} step and we propagate this velocity error step by step backward to correct the estimated velocity vector at each point along the same contour.

3. Experiments

3.1 Implementation

We applied the propagation technique to three image sequences, two of which are displayed in Figure 2. In both sequences, the object motion is in the image plane.

The propagation technique was implemented as follows:

1) Velocity vectors are first determined at a set of "corner" points in the first frame by the technique described in [6]. These corner points are marked with crosses in Figures 2 and 6.
2) The velocity vector at the corner is propagated along the contours that meet at the corner until a second corner point is encountered. The contours are followed by a very simple maximum gradient technique. A velocity vector is not computed at every pixel on the contour, but only at every k^{th} pixel, to reduce the error in α.
3) When the terminating corner point is reached, the propagation is stopped and the error velocity vector is computed. If this error is greater than a preset tolerance, then the error velocity vector is back-propagated along the same contour.

3.2 Results

The first example is a simple translation of a toy airplane (see Figure 2a). In this simple case the comparison of the velocity vectors before and after the correction processing is shown in Table 3.1. From the first and third columns of Table 3.1, it is clear that the errors are accumulated along the propagation path, and after the correction the values of V_x and V_y are very close to the accurate values (in this case, they are -1.0).

The results of the propagation procedure might depend critically on the direction of propagation - i.e., A_0 to A_n or vice versa. Experimentally, this has not been a problem. The results in Table 3.2 show the velocity vectors resulting from a "top-down" versus "bottom-up" scan of one of the contours in Figure 2a.

In the second case (Figure 2), motion consists of a translation and a rotation. The computed velocity vectors of the whole airplane and of two major parts of the airplane are shown in Figures 3,4, and 5, respectively.

Figure 6 shows a moving tool, and Figure 7 shows the velocity vectors along the main contour of this tool.

4. Conclusion

~~The velocity propagation~~ This technique described in this paper can, at least, for simple motions, reliably determine motion vectors along image contours. Although the propagation procedure was implemented as a sequential procedure which traces out contours, it is important to note that the process is not inherently sequential and can be formulated as a parallel process operating on a network of image contours.

The few examples contained in this paper all contained a single moving object. In more complex scenes, one must consider the problem of avoiding the propagation of motion vectors from one moving object to another. Also, the ability of the technique to deal with more general motions was not considered in this paper. All of these issues will be dealt with in subsequent reports.

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No.	V_x		V_y	
No.	original	corrected	original	corrected
1	-1.0*		-1.0*	
2	-1.0	-1.0	-1.0	-1.0
3	-1.0	-1.0	-1.0	-1.0
4	-1.0	-1.0	-1.0	-1.1
5	-0.9	-1.0	-0.7	-0.9
6	-1.1	-1.1	-0.9	-1.1
7	-1.0	-1.1	-0.8	-1.0
8	-0.9	-1.0	-0.7	-1.0
9	-0.9	-1.0	-0.6	-1.0
10	-1.0*		-1.0*	

Table 3.1. Velocity vector correction. The starred values are at the endpoints.

k	x	y	Vx	Vy	Vx	Vy	x	y
1	86	18	-1.0	-1.0	-1.0	-1.0	85	18
2	82	19	-0.9	-0.7	-1.0	-1.0	82	18
3	78	21	-0.9	-0.8	-1.0	-1.0	79	20
4	75	23	-0.8	-0.6	-1.0	-1.0	76	22
5	72	25	-0.9	-0.8	-1.0	-1.0	73	24
6	69	27	-0.9	-0.8	-1.0	-0.9	70	26
7	66	29	-1.0	-0.8	-1.1	-1.0	67	28
8	63	31	-0.9	-0.8	-0.7	-0.8	64	31
9	60	33	-1.0	-0.9	-1.0	-0.9	61	33
10	57	35	-1.0	-1.0	-0.9	-0.7	58	35
11	54	37	-1.0	-1.0	-0.8	-0.6	55	37
12	51	39	-1.0	-1.0	-1.0	-1.0	52	39
13	48	42	-1.0	-1.0	-1.0	-1.0	49	40
14	44	44	-1.0	-1.0	-1.0	-1.0	43	45

Table 3.2. Effects of direction of propagation. The first four columns headed x,y,vx,vy show results for one direction of propagation, and the last four columns show results for the other direction.

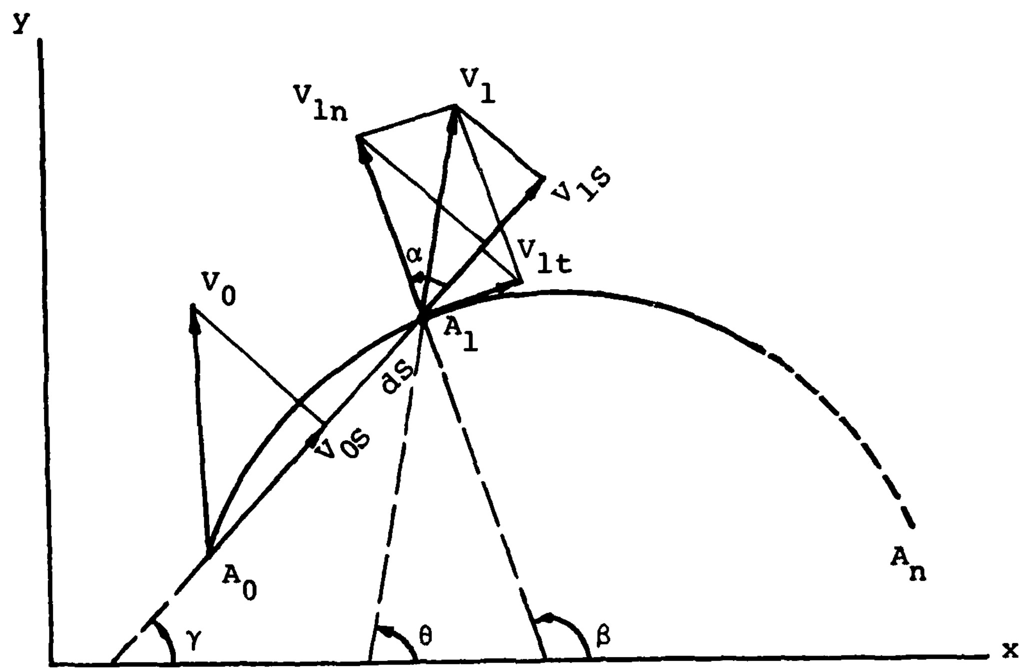
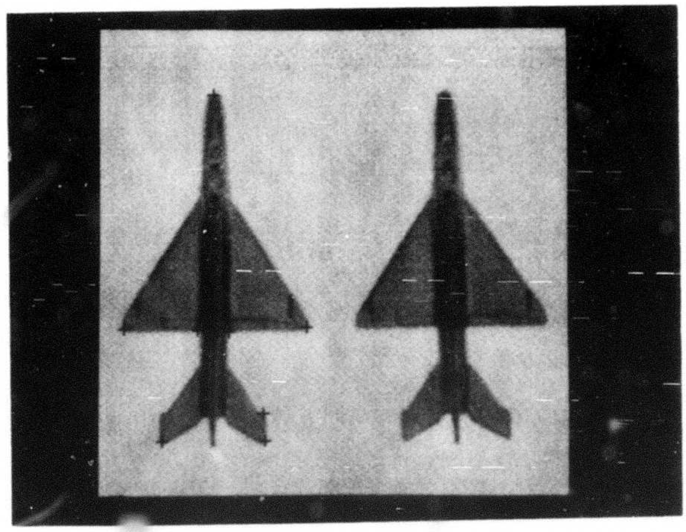
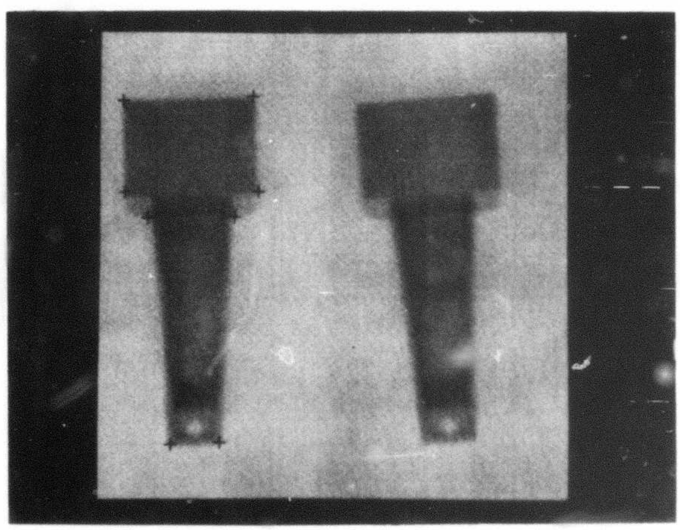


Figure 1. The geometry of the propagation along a contour in an image.



a b

Figure 2. Two frames of an airplane image.



a b

Figure 6. Two frames of a moving tool (a knife sharpener).

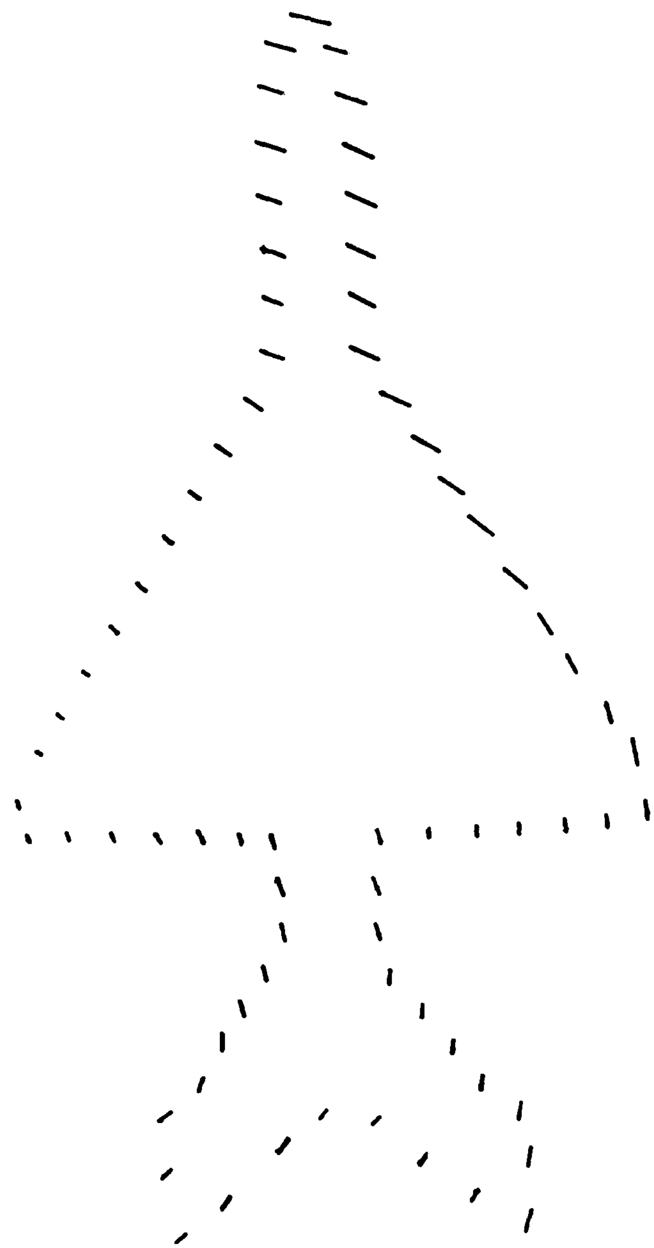


Figure 3. Velocity field using the propagation technique along the contours of the moving airplane shown in Figure 2.

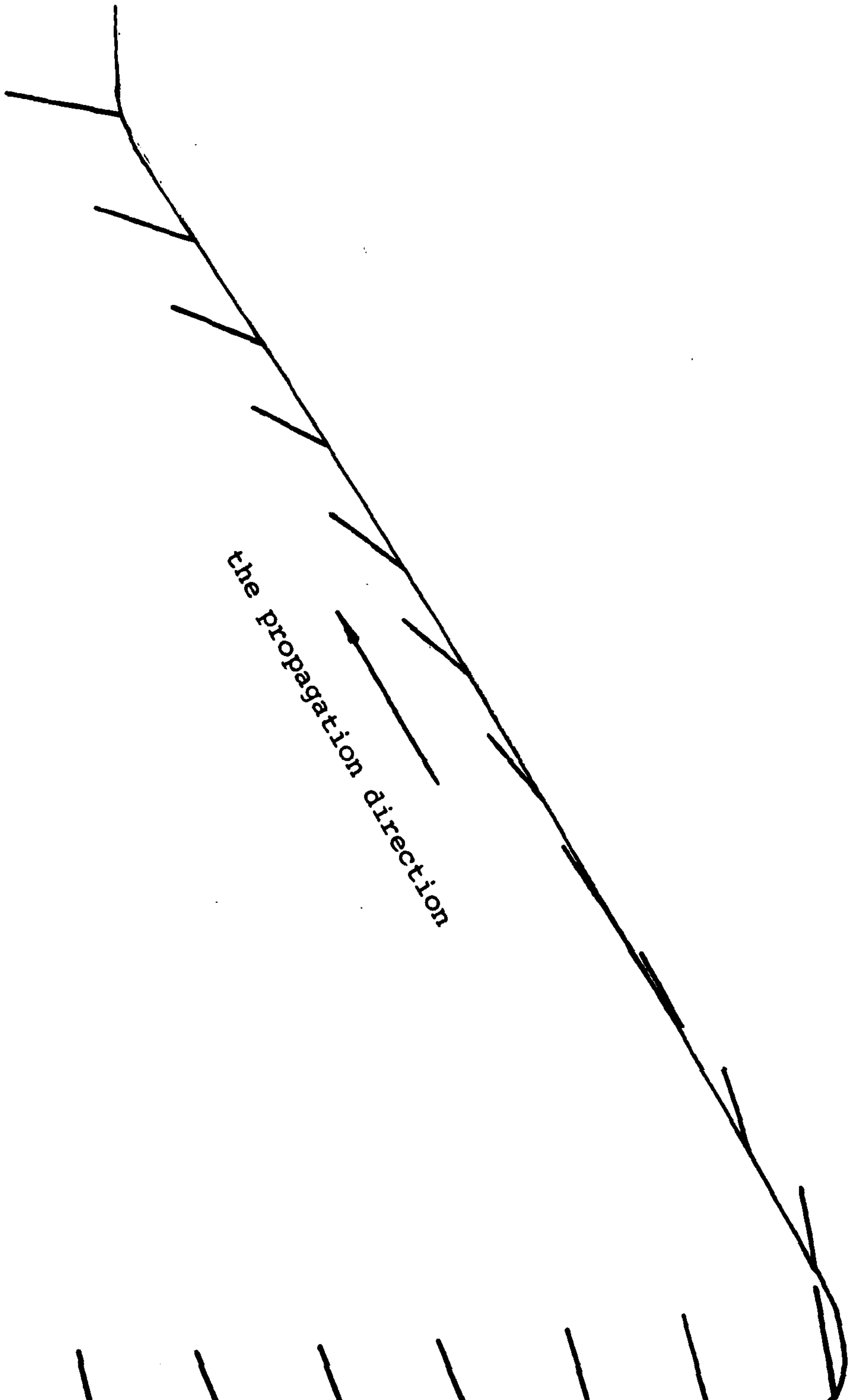


Figure 4. The enlarged velocity field of the right wing in Figure 2.

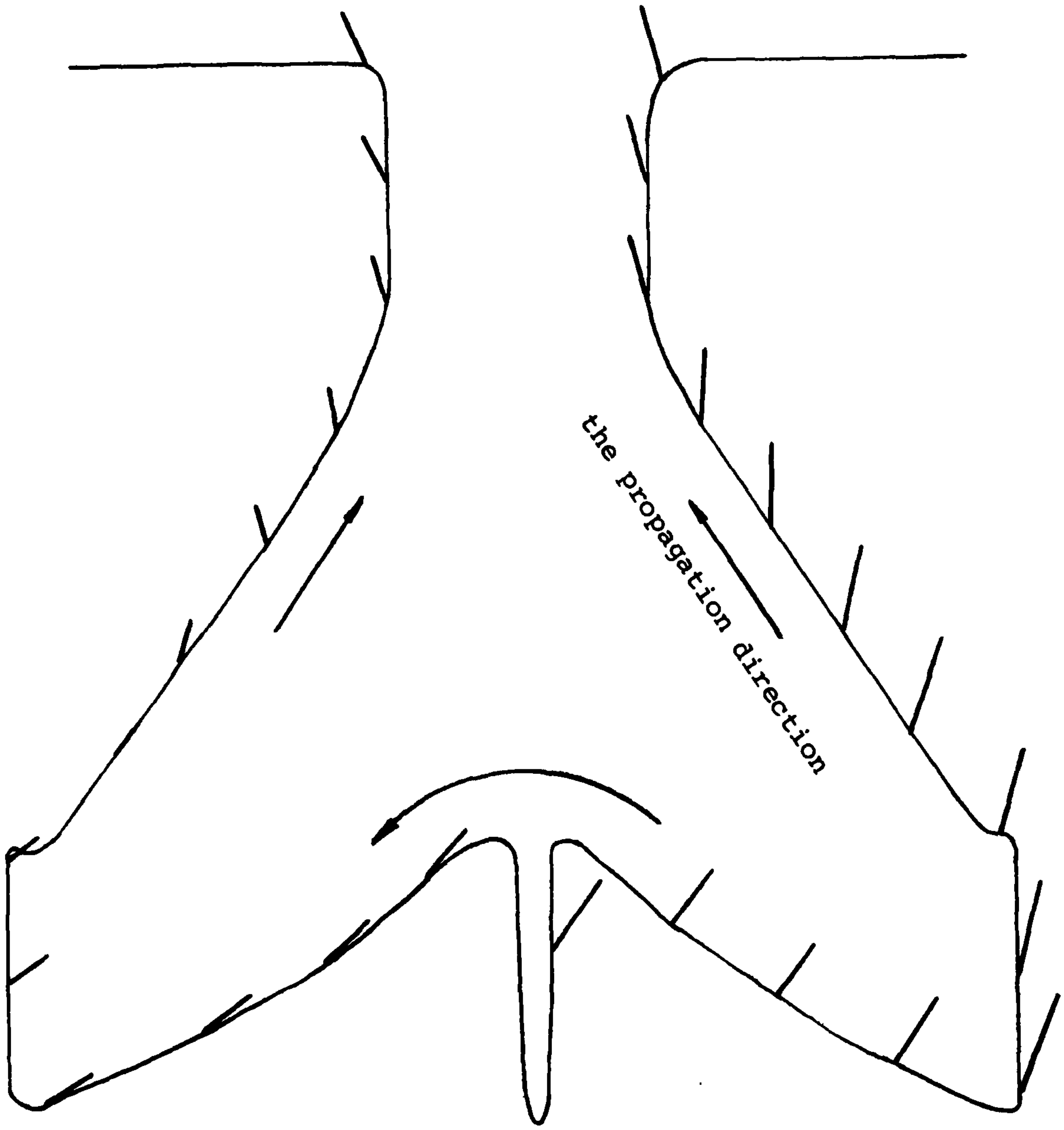


Figure 5. The enlarged velocity field of the tail in Figure 2.

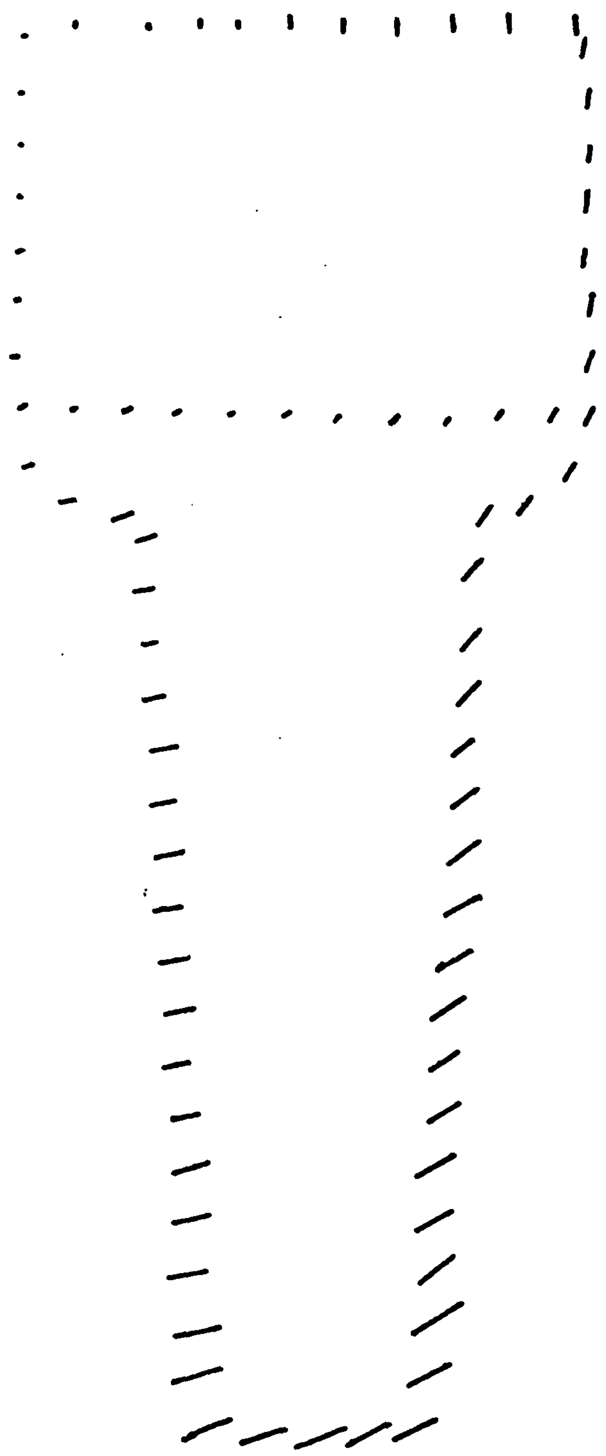


Figure 7. Velocity field of the tool.

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